Lebanese American University

Byblos

Eng. Analysis
Spring 2011
Test # 2
90 minutes

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1. Let $A = \begin{pmatrix} 2 & 3 & 0 & -1 \\ 1 & 2 & 4 & 0 \\ -2 & -2 & 8 & 2 \end{pmatrix}$

   a) Find a relation between the rows of A.
   b) Find a basis for the row-space of A.
   c) Find the rank and nullity of A.
   d) Solve the homogenous system $AX = 0$.
   e) Find a basis for the Null space of A. (25 pts)

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2. Find the adjoint of the matrix $A = \begin{pmatrix} 0 & 4 & -1 \\ -2 & 1 & 2 \\ 3 & 0 & -2 \end{pmatrix}$ (10 pts)

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3. Let $A = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$

   a) Find the inverse of A.
   b) Find a $2\times2$-matrix X satisfying $AXA - AX = I_2$.
   where $I_2$ is the identity $2\times2$ – matrix. (15 pts)

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4. Consider the system: $\begin{cases} 2x + 3y + 4z = 1 \\ x - 2y + z = 0 \\ 2y - 3z = 2 \end{cases}$

   a) Without solving, show that the system has a unique solution.
   b) Solve the system using Cramer's rule. (15 pts)

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Turn Over

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5. Let $T : R^3 \longrightarrow R^3$ be the linear transformation defined by:

$$\begin{cases} x' = 3x + 2y + z \\ y' = 4x + 4y + 3z \\ z' = 3x + 3y + 2z \end{cases}$$

Find the inverse transformation. (10 pts)

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6. Find the matrix of the reflection across the xz-plane, followed by the rotation about the x-axis through an angle of $\frac{-\pi}{3}$ (10 pts)

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7. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Let W be the set of all 2×2 – matrices X such that AX = XA.

Determine if W is a vector space or not. (15 pts)

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1. Let $A=\begin{pmatrix} 3 & 8 & 5 & 2 \\ 2 & 2 & 0 & 3 \\ 2 & 6 & 4 & 1 \end{pmatrix}$

   a) Solve the homogenous system $AX=0$, and find a basis for the Null space of A.

   b) Find a relation between the rows of A.

   c) Find the rank and nullity of A. (20 pts)

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2. Let $A=\begin{pmatrix} 6 & -4 & -3 \\ -2 & 2 & 1 \\ 3 & 1 & -2 \end{pmatrix}$ and $b=\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

   a) Using cofactors, find the inverse of the matrix A.

   b) Deduce from (a) the solution of the system $AX=b$. (15 pts)

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3. Find the eigenvalues and eigenvectors of the matrix

$$A=\begin{pmatrix} 3 & -2 & 3 \\ 1 & 0 & 3 \\ 1 & -2 & 5 \end{pmatrix}$$

(15 pts)

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4. Let $T: R^3 \longrightarrow R^3$ be the linear transformation defined by:

$$\begin{cases} x' = 3x + 5y + 5z \\ y' = 2y - 3z \\ z' = 2x + 3y + 4z \end{cases}$$

Find the inverse transformation. (10 pts)

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Turn Over

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5. Let $T_1$ be the reflection across the xz-plane, and let $T_2$ be the rotation about the y-axis through an angle of $\frac{\pi}{6}$.

   Find the matrix of:

   a) $T_1$ followed by $T_2$ using product of matrices.

   b) $T_2$ followed by $T_1$ by finding the images of the standard basis (15 pts)

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6. Let A be an invertible $n \times n$ –matrix, and let X be an $n \times n$ –matrix satisfying $2XA + AXA = A^2$

   a) Find X in terms of A.

   b) Find X if $A = \begin{pmatrix} -1 & 2 \\ 3 & -4 \end{pmatrix}$ (15 pts)

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7. Let $W = \{(x, y, x, t) \in R^4 \ / \ x - 2z = 0 \text{ and } y + z - t = 0\}$

   Determine if W is a vector space or not. (10 pts)

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